

# Parametric Spectral Correlations in Disordered and Chaotic Structures

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(Dated: February 1, 2008)

We explore the influence of external perturbations on the energy levels of a Hamiltonian drawn at random from the Gaussian unitary distribution of Hermitian matrices. By deriving the joint distribution function of eigenvalues, we obtain the  $(n,m)$ -point parametric correlation function of the initial and final density of states for perturbations of arbitrary rank and strength. A further generalization of these results allows for the incorporation of short-range spatial correlations in diffusive as well as ballistic chaotic structures.

PACS numbers: 71.23.-k, 05.45.Mt, 73.21.-b

Beginning with early pioneering work on resonances in complex atomic nuclei [1], the statistical properties of general chaotic and disordered quantum structures have come under intense scrutiny [2]. In such systems, it is conjectured (and, in the case of disordered systems, proven explicitly [3]) that the low-energy correlations of eigenvalues (and eigenfunctions) of an individual system can be inferred from the *ensemble average* properties of a random matrix distribution whose members exhibit the same fundamental symmetries [4]. For an  $N \times N$  matrix Hamiltonian  $H$  drawn from, say, a Gaussian distribution, the statistical properties of the eigenvalues  $\{\epsilon_i\}$  are completely specified by the joint distribution function

$$P(\{\epsilon_i\}) \propto |\Delta(\{\epsilon_i\})|^\beta e^{-(\beta/2) \sum_{i=1}^N \epsilon_i^2}, \quad (1)$$

where  $\Delta(\{\epsilon_i\}) = \prod_{i < j}^N (\epsilon_i - \epsilon_j)$ , and  $\beta = 1, 2$  and  $4$  for ensembles of orthogonal, unitary and symplectic symmetry, respectively. From this result, it follows that the  $n$ -point correlator of density of states (DoS)  $R_n(\{\Omega_a\}) = \langle \prod_{a=1}^n \nu(\Omega_a) \rangle$ , where  $\nu(\Omega) \equiv \bar{\Delta} \text{tr} \delta(\Omega - H)$ , can be expressed as a determinant [5]

$$R_n(\{\Omega_a\}) = \det_{ab} K_N(\Omega_a, \Omega_b). \quad (2)$$

In the Gaussian unitary ensemble (GUE) the kernel has the limiting form  $\lim_{N \rightarrow \infty} K_N(\Omega_a, \Omega_b) = \bar{k}(\Omega_a - \Omega_b)$ , where  $\bar{k}(\Omega_a - \Omega_b) = k(\Omega_a - \Omega_b) - \delta(\Omega_a - \Omega_b) \theta(b - a)$  and, defining the average energy level spacing  $\bar{\Delta}$ ,  $k(\Omega) \equiv \sin(\pi\Omega/\bar{\Delta})/(\pi\Omega/\bar{\Delta})$ . While the scaling variable  $\bar{\Delta}$  is sensitive to the specific properties of the system, the function  $k(\Omega)$  is universal provided  $\Omega \ll \mathcal{E}_c$ , where  $\mathcal{E}_c^{-1}$  is the time scale for ergodicity (equivalently,  $\Omega/\bar{\Delta} \ll g$ , where  $g = \mathcal{E}_c/\bar{\Delta}$  is the dimensionless conductance) [2].

Often one encounters applications where the interest is in the parametric dependence of the energy levels of a random Hamiltonian  $H' = H + V$  subject to an extended non-uniform (but slowly varying) perturbation  $V$  such as that imposed by a magnetic field or by an external gate voltage on a quantum dot (QD). Surprisingly, such correlations exhibit the same degree of universality: when expressed through the dimensionless parameter  $x^2 = X^2 \langle (\partial \epsilon_i / \partial X)^2 \rangle$ , where  $X$  parameterizes the strength of the external perturbation  $V$  (provided the distribution of  $H'$  is stationary with respect to  $X$ ), the

statistical properties of the entire random functions  $\epsilon_i(x)$  become universal, dependent only on the fundamental symmetries of the system [6, 7].

However, there exists a number of important applications where the perturbation is either local, or rapidly fluctuating, or both. One example is presented by the orthogonality catastrophe in a disordered conductor [8] (of volume  $\mathcal{V}$ ) where, in the coordinate basis,  $V(\mathbf{r}) = v \mathcal{V} \delta^d(\mathbf{r} - \mathbf{r}_0)$ . Within random matrix theory (RMT), this is modeled by a rank  $r = 1$  perturbation  $V_{ij} = v N \delta_{ij} \delta_{i1}$ . The shape-distorting effect of a gate acting on a ballistic QD can be either global or local, depending on the relation between the spatial extent of the gate and the wavelength of the electron  $\lambda_F$ . Similarly, attempts have been made to explore the pumping of electrons through a ballistic quantum dot under the action of two local gates [9] which effectively represent a rank  $r = 2$  perturbation. The potential  $V(\mathbf{r}) = v \mathcal{V} [\delta^d(\mathbf{r} - \mathbf{r}_1) - \delta^d(\mathbf{r} - \mathbf{r}_2)]$  imposed by a bistable impurity in a disordered metallic host presents another application of a rank  $r = 2$  perturbation. A bistable dislocation segment provides an example of a perturbation of a still higher rank. The freedom to explore perturbations of high rank is easily afforded in experiments on microwave resonators with movable scatterers [10], or in STM devices where the tip does not resolve the Fermi wavelength.

The aim of this letter is to show that, when applied to a member of the unitary random matrix ensemble, perturbations of arbitrary rank  $r$  can be incorporated into a general scheme which allows analytical expressions for the joint distribution function and the DoS correlation function to be determined explicitly. In particular, for a rank  $r$  matrix  $V = N \hat{v}$  whose non-zero eigenvalues form a set  $N \{v_k\}_{k=1}^r$ , the  $(n,m)$ -point parametric correlation function between the initial and the final DoS  $R_{nm}(\{\Omega_a\}, \{\Omega'_b\}) = \langle \prod_{a=1}^n \nu(\Omega_a) \prod_{b=1}^m \nu'(\Omega'_b) \rangle$  can be expressed as the  $(n+m) \times (n+m)$  determinant

$$\lim_{N \rightarrow \infty} R_{nm}(\{\Omega_a\}, \{\Omega'_b\}) = \det \begin{pmatrix} \bar{k}(\Omega_a - \Omega_b) & \hat{\mathcal{D}}_{\hat{v}, \Omega_a}^{-1} \tilde{k}(\Omega_a - \Omega'_{b'}) \\ \hat{\mathcal{D}}_{\hat{v}, \Omega'_a} k(\Omega'_{a'} - \Omega_b) & \bar{k}(\Omega'_{a'} - \Omega'_{b'}) \end{pmatrix}. \quad (3)$$

Here  $\tilde{k}(\Omega_a - \Omega'_{b'}) = k(\Omega_a - \Omega'_{b'}) - \delta(\Omega_a - \Omega'_{b'})$ , and

$$\hat{\mathcal{D}}_{\hat{v},\Omega} = \det \left( \mathbb{1} - \hat{v} \frac{d}{d\Omega} \right) \equiv \prod_{k=1}^r \left( 1 - v_k \frac{d}{d\Omega} \right), \quad (4)$$

while the inverse of  $\hat{\mathcal{D}}_{\hat{v},\Omega}$  has a convenient integral representation

$$\hat{\mathcal{D}}_{\hat{v},\Omega}^{-1} h(\Omega) = \int \prod_{k=1}^N (d\chi_k d\chi_k^*/\pi) e^{-|\chi|^2} h(\Omega + \chi^\dagger \hat{v} \chi).$$

The diagonal blocks of (3) reproduce the standard correlations (2) (see also Ref. [11]), while, in the particular case of rank  $r = 1$ , for  $n = m = 1$  Eq. (3) recovers the result obtained in Ref. [8]. The corresponding joint distribution function of the combined set of eigenvalues  $\{\epsilon_i\}$  of the matrix  $H$  and  $\{\epsilon'_i\}$  of  $H'$  is given by

$$\mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\}) \propto \mathcal{P}(\{\epsilon_i\}) \frac{\Delta(\{\epsilon'_i\})}{\Delta(\{\epsilon_i\})} \det_{ij} \left[ \hat{\mathcal{D}}_{\hat{v},\epsilon_i}^{-1} \delta(\epsilon_i - \epsilon'_j) \right] \quad (5)$$

where  $\mathcal{P}(\{\epsilon_i\})$  represents the distribution function (1) of the eigenvalues of the matrix  $H$  for  $\beta = 2$ .

To apply these results to physical systems, it is necessary to effect a generalization to account for cases where the Hamiltonian dynamics  $H$  is generated by a sum of regular and random components. This leads to non-vanishing off-diagonal terms in the ensemble average propagator  $\langle G(\epsilon_F) \rangle = \langle (\epsilon_F - H)^{-1} \rangle$ . In disordered metals, the corresponding non-universal terms are known as Friedel oscillations and reflect the short-ranged ballistic nature of wave propagation via the Friedel function  $f(\mathbf{r}) = (\bar{\Delta}V/\pi) \text{Im} \langle G(\epsilon_F - i0; 0, \mathbf{r}) \rangle$ . In the presence of an assembly of rank one perturbations  $V(\mathbf{r}) = \sum_k v_k \delta^d(\mathbf{r} - \mathbf{r}_k)$ , parametric spectral correlations are heavily influenced by the spatial arrangement of the wavefunction around the impurities. Surprisingly, the correlation function in this case retains the overall structure of (3), and the effect of the Friedel oscillations is incorporated into the expression for  $R_{nm}$  by means of the generalization

$$\hat{\mathcal{D}}_{\hat{v},\Omega} \mapsto \hat{\mathcal{D}}_{\hat{f}\hat{v},\Omega} \equiv \det \left( \mathbb{1} - \hat{f} \hat{v} \frac{d}{d\Omega} \right), \quad (6)$$

where the elements of the  $r \times r$  matrix  $\hat{f}$  are prescribed by the Friedel function  $f_{kl} \equiv f(\mathbf{r}_k - \mathbf{r}_l)$ , and  $\hat{v}$  is the diagonal matrix with entries  $v_k \delta_{kl}$ . If the points  $\mathbf{r}_k$  and  $\mathbf{r}_l$  are separated by a distance in excess of the mean-free path,  $f(\mathbf{r}_k - \mathbf{r}_l) \mapsto \delta_{kl}$ , and Eq. (4) is recovered.

Within RMT, Eq. (3) applies to perturbations of arbitrary rank and strength. In systems where the Hamiltonian dynamics includes a regular component (such as diffusive or ballistic conductors) Eqs. (3) and (6) represent the leading (zeroth) order term in an expansion in the inverse powers of the dimensionless conductance  $g$ . Whether the subleading terms in this expansion are

small depends sensitively on the range and profile of the perturbation. If  $\text{tr}(\hat{f}V)^2 > \text{tr}(V\Pi V)$ , where  $\Pi$  is the diffusion (Perron-Frobenius) propagator in diffusive (ballistic) systems, the results above apply without qualification [12, 13]. In the opposite limit, when rescaled by a single parameter which incorporates diffusive correlations at length scales in excess of the mean free path, parametric correlations acquire a universal form [6], independent of the detailed properties of the system. The structure of these correlations can be inferred from the RMT above: for an extended perturbation of rank  $r \sim O(N)$ , the requirement that  $\hat{\mathcal{D}}$  stays finite as  $N \rightarrow \infty$  implies that  $\text{tr} V^2 \sim O(N^2 \bar{\Delta}^2)$ . It follows that, for all  $n \geq 3$ ,  $\text{tr}(V/N\bar{\Delta})^n \rightarrow 0$ , and  $\hat{\mathcal{D}}_{\hat{v},\Omega} \rightarrow \exp[-(\text{tr} \hat{v}^2/2) d^2/d\Omega^2]$  reproducing identically the results of Ref. [6].

In the standard Wigner-Dyson RMT, the determinantal structure (2) of the many-point correlation functions makes it possible to obtain analytical results for properties associated with *individual* energy levels, such as level spacing statistics and its various generalizations [5]. Similarly, we anticipate that on the basis of the  $(n, m)$ -point parametric correlation function obtained in this Letter, an analogous theory of ‘parametric level spacings’ in the GUE can be developed.

To illustrate this point, we consider the evolution of a single level under the influence of a rank  $r = 1$  perturbation. Specifically, we study the distribution  $P(d) = \bar{\Delta} \langle \sum_i \delta(\Omega - \epsilon_i) \delta[\Omega + d - \epsilon'_i(v)] \rangle$  of distances  $d = \epsilon'_i(v) - \epsilon_i$  between a given level  $\epsilon_i$  and its ‘descendant’  $\epsilon'_i(v)$ . This quantity is of independent interest in the context of Coulomb blockade (CB) peak spacings in disordered QDs. It is known that RMT-based statistical theory of peak spacings shows poor agreement with experiment [14]. The disagreement is most likely attributable to the effects of strong Coulomb interaction which manifest themselves as ‘spectral scrambling’ [15] – readjustment of single-particle eigenstates to accommodate the non-uniform spatial distribution of the charge of an added electron, and the parametric ‘gate effect’ [16] – distortion of the shape of the QD as the gate voltage is changed to sweep through a sequence of CB peaks. It would be interesting to study the latter effect in isolation, for example by comparing the two sequences of CB peaks corresponding to  $v = 0$  and  $v \neq 0$ . If the ‘gate effect’ obeys RMT statistics, the distribution of differences in peak positions should coincide with  $P(d)$ . An  $r = 1$  perturbation can be simulated, for example, by adjusting the voltage on a finger-shaped gate whose tip has a spatial extent smaller than  $\lambda_F$ .

A particular feature of the  $r = 1$  perturbation is the identity  $\epsilon_i < \epsilon'_i < \epsilon_{i+1}$  [8], where the levels are numbered in increasing order (assuming  $v > 0$ ).  $P(d)$  thus coincides with the distribution  $\tilde{P}(d)$ , defined as the probability that an interval of length  $d$  is bounded by a pair  $(\epsilon_i, \epsilon'_j(v))$  at its ends, and does not contain inside it any levels from either of the two sets. The above identity ensures  $j = i$ . The standard method of computing level spacing distributions in RMT [5] affords a straightfor-

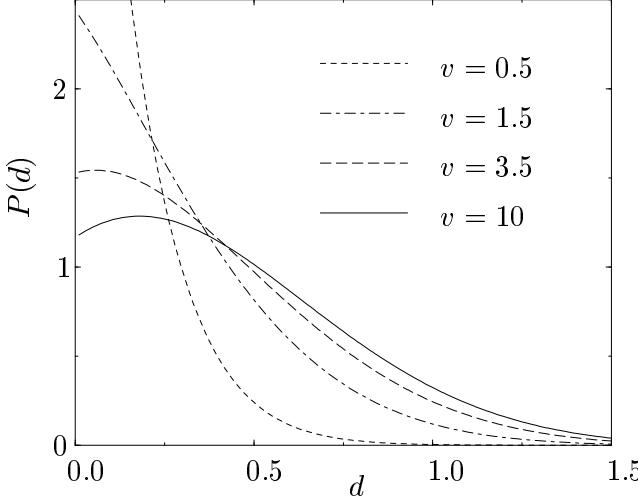


FIG. 1: The  $P(d) \equiv \tilde{P}(d)$  curves at several characteristic values of  $v$  for a rank  $r = 1$  perturbation.

ward generalization to the parametric case allowing the distribution  $\tilde{P}(d)$  to be expressed as

$$\tilde{P}(d) = \det(\mathbb{1} - K) [\mathcal{G}_{11}(0,0)\mathcal{G}_{22}(d,d) - \mathcal{G}_{12}(0,d)\mathcal{G}_{21}(d,0)]$$

(cf. Eq. (5.4.31) of Ref. [5]), where the integral kernel

$$K(\epsilon, \epsilon') = \begin{pmatrix} k(\epsilon - \epsilon') & \hat{\mathcal{D}}_{\hat{v}, \epsilon}^{-1} \tilde{k}(\epsilon - \epsilon') \\ \hat{\mathcal{D}}_{\hat{v}, \epsilon} k(\epsilon - \epsilon') & k(\epsilon - \epsilon') \end{pmatrix}$$

is defined on the segment  $[0, d]$  of the real line, and  $\mathcal{G}_{\alpha\beta}(\epsilon, \epsilon')$  is the inverse of  $\mathbb{1} - K^{-1}$ . In Fig. 1 we present a set of  $\tilde{P}(d)$  curves at several different values of  $v$ . Note that in the limit  $v \rightarrow 0$  the distribution  $\tilde{P}_{v \rightarrow 0}(d) = \frac{1}{v} e^{-d/v}$  coincides with the appropriately scaled level velocity distribution [17], while for  $v \gtrsim 3$  the curves  $\tilde{P}(d)$  develop a maximum at a finite value of  $d$  signifying weak repulsion between the old and the new levels.

Let us turn now to the derivation of Eqs. (3), (4), and (6). To obtain the correlation function  $R_{nm}$  one can fol-

low at least two different routes: By implementing conventional methods of statistical field theory,  $R_{nm}$  can be expressed in the form of a supersymmetric field integral with an action of non-linear  $\sigma$ -model type [3]. Although such an approach has the advantage of allowing a systematic development of non-universal corrections in both the disordered and chaotic environments, this method is largely tailored to the consideration of two-point correlation functions (i.e.  $n = m = 1$ ). Thus, we will follow a complementary route employing a method based on orthogonal polynomials [5]. A similar approach was employed in Ref. [18] to study the statistics of eigenvalues of matrices belonging to the GUE under the influence of a finite rank non-Hermitian perturbation. The latter ensemble serves as a model for describing resonances in quantum chaotic scattering [19].

The joint distribution of complex  $N \times N$  Hermitian matrices  $H$  and  $H'$  is defined by

$$\mathcal{P}(H, H') = \mathcal{P}(H) \delta(H' - H - V), \quad (7)$$

where  $\mathcal{P}(H)$  represents the Gaussian distribution function of  $H$ , and the matrix  $\delta$ -function is understood as a product of  $N^2$  scalar  $\delta$ -functions (one per each independent component of the complex Hermitian matrix  $H$ ). Setting  $H = U \hat{\epsilon} U^\dagger$  and  $H' = U' \hat{\epsilon}' U'^\dagger$  where  $\hat{\epsilon}$  and  $\hat{\epsilon}'$  represent the diagonal matrices of eigenvalues, the joint distribution function  $\mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\})$  is obtained from (7) by integrating out the ‘angular’ degrees of freedom,

$$\mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\}) = \int_{\mathrm{U}(N)} \Delta^2(H) d\mu(U) \int_{\mathrm{U}(N)} \Delta^2(H') d\mu(U') \mathcal{P}(H, H'),$$

where  $d\mu(U)$  represents the invariant Haar measure on the unitary group  $\mathrm{U}(N)$ , and  $\Delta(H) \equiv \Delta(\{\epsilon_i\})$ . To facilitate the angular integration it is convenient to introduce a Lagrange multiplier in the form of the Hermitian matrix  $\Lambda = U_\Lambda \hat{\lambda} U_\Lambda^\dagger$ , where  $\hat{\lambda}$  represents the diagonal matrix of eigenvalues of  $\Lambda$ , after which the expression for the joint distribution function can be brought to the form

$$\mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\}) \propto \int_{\mathrm{U}(N)} \Delta^2(H) d\mu(U) \int_{\mathrm{U}(N)} \Delta^2(H') d\mu(U') \prod_{i=1}^N \int_{-\infty}^{\infty} d\lambda_i \int_{\mathrm{U}(N)} \Delta^2(\Lambda) d\mu(U_\Lambda) e^{i\mathrm{tr}[\Lambda(H+V-H')]-\mathrm{tr}H^2}.$$

The overall numerical constant can be restored by demanding the correct normalization. Integration over  $U$  and  $U'$  is performed by means of the Itzykson-Zuber integral [20]

$$\int_{\mathrm{U}(N)} d\mu(U) e^{i\mathrm{tr}(AUBU^\dagger)} \propto \frac{\det[\exp(ia_i b_j)]}{\Delta(A)\Delta(B)},$$

where  $\{a_i\}$ , and  $\{b_i\}$  represent, respectively, the eigenvalues of the  $N \times N$  Hermitian matrices  $A$  and  $B$ . As a result,

$$\mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\}) \propto \mathcal{P}(\{\epsilon_i\}) \frac{\Delta(\{\epsilon'_i\})}{\Delta(\{\epsilon_i\})} \int_{\mathrm{U}(N)} d\mu(U_\lambda) \prod_{i=1}^N \int_{-\infty}^{\infty} d\lambda_i e^{i\mathrm{tr}(\Lambda V)} \sum_{\mathrm{PP}'} (-1)^{\mathrm{P}+\mathrm{P}'} \exp \left[ i \sum_{i=1}^N (\epsilon_i \lambda_{\mathrm{P}(i)} - \epsilon'_i \lambda_{\mathrm{P}'(i)}) \right], \quad (8)$$

where the sum over  $\mathrm{P}$  runs over all permutations of the indices  $1, \dots, N$ , and  $(-1)^{\mathrm{P}}$  denotes the signature of the permutation. However, using the Itzykson-Zuber formula to undertake the remaining angular integration over  $U_\Lambda$  is impractical because of the  $N - r$  degenerate (zero) eigenvalues of the perturbation  $V$ . Instead, it is convenient to express the rank  $r$  perturbation in the form  $V = N \sum_{k=1}^r v_k \mathbf{a}_k \otimes \mathbf{a}_k^\dagger$  where  $\mathbf{a}_k$  form a set of  $r$  complex mutually orthogonal  $N$ -component vectors of unit length. Due to the invariance of the measure  $\mu(U)$ , the integral in Eq. (8) does not depend on  $\mathbf{a}_k$ , and therefore its value does not change upon averaging over them. Although the measure  $d\mathcal{W}$  of the integral over  $\mathbf{a}_k$  must formally enforce the conditions of normalization and mutual orthogonality, in the large- $N$  limit it can be replaced by the product of the Porter-Thomas [21] distributions of their components:

$$d\mathcal{W}[\{\mathbf{a}\}] = \prod_{k=1}^r \prod_{i=1}^N \frac{N da_{ki} da_{ki}^*}{\pi} e^{-N|a_{ki}|^2}.$$

Absorbing the rotations of  $\Lambda$  into the invariant measure  $d\mathcal{W}$ , and integrating out the eigenvalues of  $\Lambda$ , one obtains

$$\begin{aligned} \mathcal{P}(\{\epsilon_i\}, \{\epsilon'_i\}) &\propto \mathcal{P}(\{\epsilon_i\}) \frac{\Delta(\{\epsilon'_i\})}{\Delta(\{\epsilon_i\})} \sum_{\mathrm{PP}'} (-1)^{\mathrm{P}+\mathrm{P}'} \\ &\times \int d\mathcal{W}[\{\mathbf{a}\}] \prod_{i=1}^N \delta \left( \epsilon'_{\mathrm{P}'(i)} - \epsilon_{\mathrm{P}(i)} - N \sum_{k=1}^r v_k |a_{ki}|^2 \right). \end{aligned} \quad (9)$$

Then, introducing new variables  $\chi_{ki} = \sqrt{N} a_{ki}$ , the integral over  $\chi_{ki}$  is identified as the integral representation of  $\prod_{i=1}^N \hat{\mathcal{D}}_{\hat{v}, \epsilon_i}^{-1}$ , thus establishing Eq. (5).

To obtain the correlation function  $R_{nm}$  the joint distribution function (9) is multiplied by  $\prod_{a=1}^n \sum_{i_a=1}^N \delta(\Omega_a - \epsilon_{i_a}) \prod_{b=1}^m \sum_{j_b=1}^N \delta(\Omega'_b - \epsilon'_{j_b})$  and then integrated over all eigenvalues  $\{\epsilon_i\}$  and  $\{\epsilon'_i\}$ . To perform the integrals over  $\{\epsilon_i\}$  we utilize the following identity:

$\prod_{i=1}^N \hat{\mathcal{D}}_{\hat{v}, \epsilon_i}^{-1} \Delta(\{\epsilon_i\}) = \Delta(\{\epsilon_i\})$ . Upon integrating by parts over all  $\epsilon_i$  except the  $n$  fixed energies  $\epsilon_{i_a}$ , and integrating over all  $\epsilon'_j$ , the correlation function can be recast as [22]

$$\begin{aligned} R_{nm}(\{\Omega_a\}, \{\Omega'_b\}) &\propto \sum_{\substack{i_1 \dots i_n \\ j_1 \dots j_m}} \int \prod_{i=1}^N e^{-\epsilon_i^2} d\epsilon_i \\ &\times \left[ \left( \prod_{a=1}^n \delta(\epsilon_{i_a} - \Omega_a) \hat{\mathcal{D}}_{\hat{v}, \epsilon_{i_a}} \right) \Delta(\{\epsilon_i\}) \right] \\ &\times \left[ \left( \prod_{a=1}^n \hat{\mathcal{D}}_{\hat{v}, \epsilon_{i_a}}^{-1} \right) \Delta(\{\epsilon_i\}) \prod_{b=1}^m \delta(\epsilon_{j_b} - \Omega'_b) \right]. \end{aligned} \quad (10)$$

After some algebra, Eq. (3) is straightforwardly established following the standard procedure of expanding the Vandermonde determinants in terms of Hermite polynomials and using their orthogonality properties.

Finally, to establish the form of  $\hat{\mathcal{D}}$  in the case when Friedel correlations are present, we note that  $V_{ij}$  can be written as  $\sum_{lp} U_{il} [\sum_{k=1}^r v_k \psi_k^{(l)} \psi_k^{(p)\dagger}] U_{pj}^\dagger$ , where  $\psi_k^{(l)}$  is the  $k$ -th component of the  $l$ -th eigenfunction of  $H$ . To the leading order in  $1/g$ , the distribution of  $\psi_k^{(l)}$  is uncorrelated with  $\{\epsilon_i\}$ , and is given by Berry's conjecture [23, 24]. In particular, in the unitary ensemble,

$$d\mathcal{W}[\{\psi\}] \propto \prod_{kl} d\psi_k^{(l)*} d\psi_k^{(l)} e^{-\sum_l \psi_k^{(l)*} (\hat{f}^{-1})_{kk'} \psi_{k'}^{(l)}}.$$

Absorbing  $U$  into  $U_\lambda$  and reversing the order of integration over  $d\mathcal{W}$  and  $U'$ , we arrive at Eqs. (5), (6).

To summarize, we have demonstrated that the  $(n, m)$ -point parametric correlation function of the DoS in GUE possesses a determinantal structure for perturbations of arbitrary rank. Beyond GUE, a generalization of this determinantal structure is shown to accommodate spatial correlations induced by ballistic wave propagation at length scales shorter than the mean free path.

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